CHAOS THEORY

Applications to population systems

Ben Newcomer
April 1999

Word Count [excluding figures]: 1570

INTRODUCTION

Chaos is everywhere—chaos, that apparently random disorder wherein lies order. Chaos, the fodder of techno thrillers in the bookstore. Chaos, the inspiration for revised studies of the human mind, the stock market, and weather systems. Few people would argue that the 20th Century was witness to countless important discoveries, though some would argue what the *most* important discoveries were. Quantum theory and Einstein's theory of relativity would by most authorities' standards be considered two of the most important foundations laid during the 20th Century; chaos theory is beginning to be seen as a third party to add to the category. Chaos theory developed out of attempts to model weather that revealed the role of sensitive dependence in such systems and led the way to more complex ideas about the behavior of complex systems. Simply put, chaos theory looks at the behavior of nonlinear dynamic[1] systems and describes it in mathematical terms. One characteristic which chaotic systems exhibit is that of a sensitive dependence on initial conditions, the classic example of this being a butterfly flapping its wings in Tokyo causing a thunderstorm in New York; a more realistic example of this might be the development of a cold front where computer models did not predict one. Both of these examples illustrate quite well the property of sensitive dependence possessed by all chaotic systems. It is important to note, however, that a system exhibiting this sensitive dependence need not be chaotic. This quality of sensitive dependence means that all chaotic systems are deterministic in the strictest mathematical sense, as knowledge of exact initial conditions could predict *exactly* how a system would evolve. However, this sensitivity to *exact* conditions is so great that the final state reached by a chaotic system is unpredictable in practical real-world terms, a difficulty that makes the behavior of the entire system indistinguishable from random processes. In

[1] Sometimes referred to as "dynamic***al***"

studying chaos, especially on a very basic level, fractals inevitably become a topic of discussion, not because chaos theory is comprised only of fractals but because fractals are an interesting discovery of chaos theory which help to illustrate its characteristics. Fractals are created through iterated functions, which essentially take a number, use that number as input, and then use the output as a new input, that output as the next input, etcetera, repeated ad infinitum. Fractals are fascinating objects, wonderfully educational tools, and in some cases even helpful objects for modeling certain systems, but will not be focused on in this paper. This paper will focus on chaos theory and how it can be applied in modeling biological systems, focusing on its applications to population systems.

USING CHAOS THEORY TO MODEL POPULATIONS

Today's massive computing power offers unique opportunities to study interactions within incredibly complex systems, yet there still exists a deceptively simple and innocuous question of how single populations behave over time. It seems intuitive that population systems would not follow completely predictable paths due to the sheer complexity and number of variables at work in the "real-world" scenario. Two basic approaches are taken to the topic of population dynamics: one, that populations are regulated steadily with exceptions; and the other, that populations are erratic, with exceptions. Traditional scientific philosophy would tend to dictate that deviations from periodicity are typically caused by some sort of outside interference or a flaw in the experiment (Gleick 68). In population dynamics as in virtually any other system study, this translates to interpreting a complicated pattern of data as random noise interspersed within a periodic fluctuation. Chaos theory, however, provides new insight into the ostensibly random "environmental noise" (as these fluctuations are currently being widely disregarded as) found in population systems, providing us with a better opportunity to understand cycles observed within populations.

Biologist Robert May is generally noted as the first researcher to apply principles of chaos theory to population systems (Houghton). May used a very simple logistic equation of $X_{next}=RX(1-X)$ to represent an imaginary population of fish, where X represented the number of fish in the population and R was a parameter on which the population growth was dependent. What May found was that changing the parameter had a drastic impact on the final outcome in his population of fish. With a very low R in May's equation, the population eventually reaches a steady state (*see Fig 1*). With a slightly higher R value, the equation produces a system that oscillates in equilibrium, never settling down to a steady state—a model which might describe a two-year cycle found within a population. What May found as he increased the growth rate parameter, however, was that the nonlinearity of the system increased (Houghton). Not only was the final character of the system's equilibrium determined by the growth rate parameter, but whether or not the system would ultimately reach an equilibrium state was in fact determined by this parameter (Houghton). With the regular cycle, the system can be said to be settled in a regular, periodic "orbit"; once the system becomes chaotic, however, the orbit also becomes chaotic, the function describing a population which never returns to a point visited before. With a growth-rate parameter of four in May's simple logistic equation, the system that was initially periodic becomes chaotic (*see Fig 4*), and anyone observing a population governed by this equation would assume that the changes inherent in the system were brought about by completely random external "noise" (Gleick 73). In fact, research has been done which indicates that chaos might be present even in orderly, "period three" systems (period three denoting a regular oscillation between three levels) (Gleick 69).

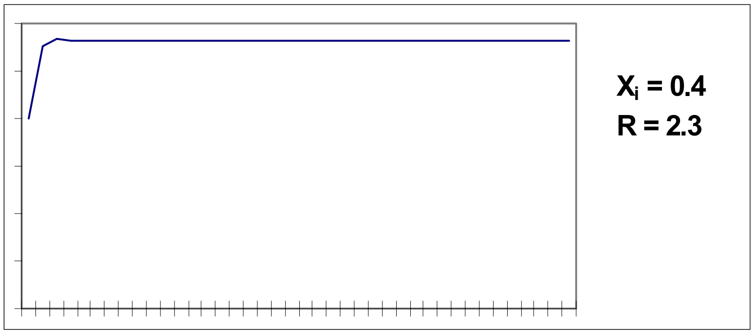

Figure 1: May's imaginary fish population as governed by a very low parameter R eventually settles down to a steady state.

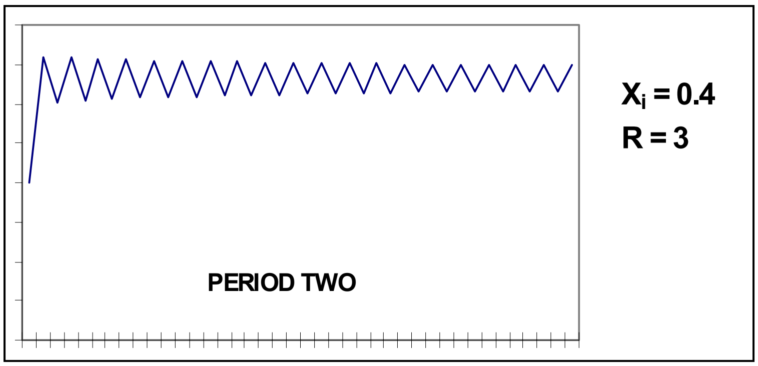

Figure 2: Given a slightly higher R value, the fish population oscillates between two values—mirroring a two-year cycle—but never settles down to a steady state as in the initial plot.

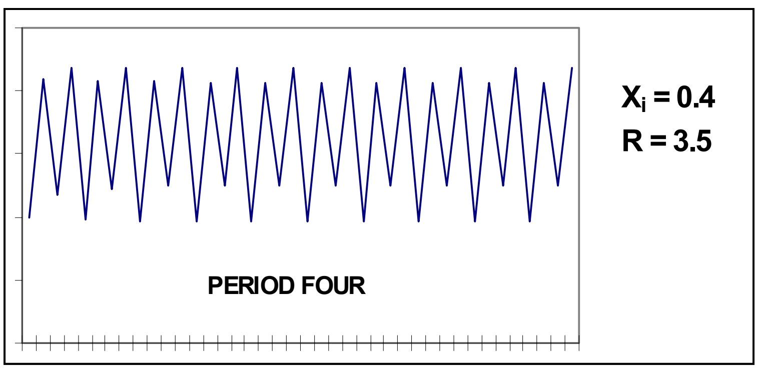

Figure 3: Raising the parameter increases the cycle found in the system to repeating every four years.

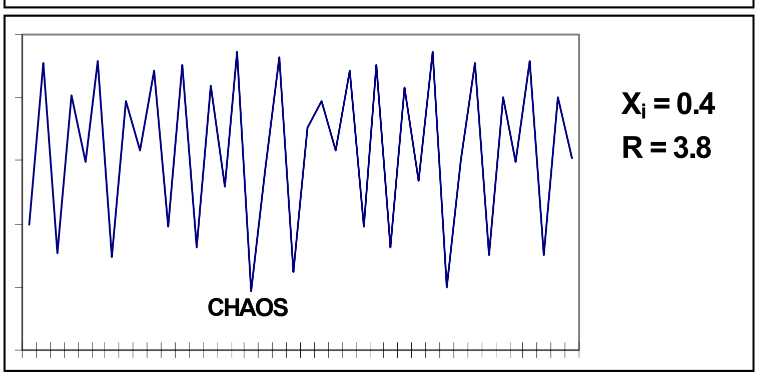

Figure 4: With a higher growth-rate parameter yet, the system converts from periodicity to chaos.

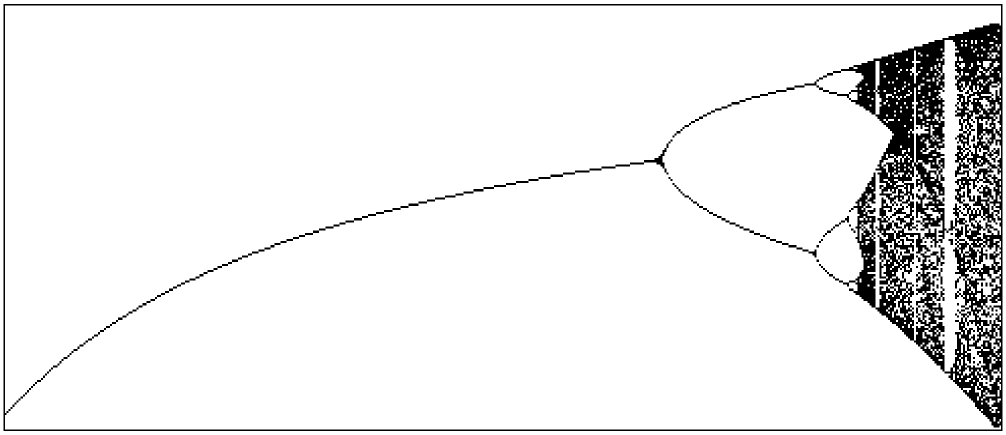

Figure 5 - Bifurcation Diagram: R (the growth-rate parameter) is represented on the horizontal axis (increasing from left to right from 1 to 4), and the final population of the fish is represented on the vertical axis. The first branching is the bifurcation where the steady state of the system converts to a 2-year cycle, oscillating between two values. Shaded areas denote values for R where the system is chaotic. (diagram Shibata)

To describe the changes in state of the system, May developed a “bifurcation diagram,” which simply and graphically illustrated the relationship between the growth-rate parameter and the behavior of the population as a system. In this particular case, the bifurcation diagram (now a common tool) plotted final population after a set time on the horizontal axis against the growth-rate parameter on the vertical axis (*see Fig. 5*) The graph was called a “bifurcation diagram” for the forks in the graph where the overall qualitative behavior of the system changed (e.g., changed from ending up in a steady state to a period two state, oscillating between two values in equilibrium) Even more bizarre than these bifurcations is the fact that windows of periodicity can be found in the chaos as the growth rate increases further; that is, as the tendency toward nonlinearity increases, windows of periodicity appear (Gleick 71).

Houghton notes that many population systems do not have independent growth rate parameters which are equivalent to the rates used in May’s equation that induced the chaotic behavior. More importantly, however, many population systems still exhibit properties of a chaotic system despite having initial growth rate parameters which would be thought to induce the behavior (Houghton). One hypothesis for this behavior is that the impact of external ecological stimuli of increasing growth rate must be taken into account, as it would account for the chaotic properties of many population systems (Houghton). One result of this view would be that shifts in certain variables (e.g., survival) could be interpreted as causing an overall shift in behavior of population system—for example, a change in food available could not only affect the population level of a species directly but also cause a shift from a two-year cycle to a three-year cycle ("Population" - *see Fig. 6*). Using this viewpoint could be extremely beneficial in helping to understand the behavior of populations under stress, and a better forecast could be made for the apparent recovery or depopulation of a certain species ("Population").

Ludwig et al. note that it is not entirely plausible to assume an infinite number of possible behaviors for a single population system; however, their research suggests that concepts of chaos theory can be applied to population systems in order to describe scenarios in which a system may change from having a single stable equilibrium to having several (though not infinite) stable equilibria. This theory of multiple stable states of population systems is gaining increasing validity with more studies which show systems in the coral reefs, African rangelands, and grasslands possessing multiple states of qualitative if not quantitative cycles (Ludwig et al.)

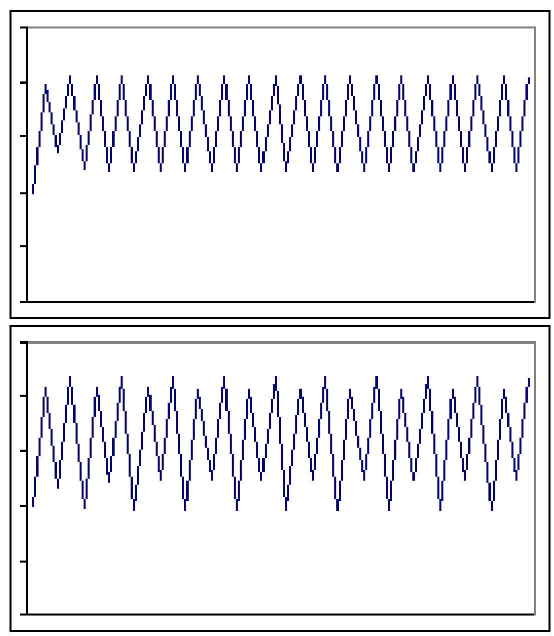

Figure 6 - The above graph would model a population system in a 2-year cycle; following a chaos-theory model, an environmental stimulus (e.g., famine) could cause the system to bifurcate, or shift to the behavior exhibited in the lower graph of a 3-year cycle.

CONCLUSION

As a better and more complete understanding of the implications of chaos theory is gained, so changes our understanding of the systems around us. Much research has been done with the mindset that minor fluctuations within a system can be ignored and attributed to external "noise"; what chaos theory is hinting at is that these "minor" fluctuations may in fact not be due to noise but intrinsic to the systems. And indeed, chaos theory is providing valuable insight to the behavior of population systems thanks to the pioneering work of Robert May with a simple population model using a logistic equation. This deceptively simple groundwork has led the way to a much more complex understanding of the interactions at work in population systems. Chaos theory has provided a means of understanding how the long-term effects of an environment (or other) stimulus might be not to effect an overall decrease or increase in population but to change the nature of fluctuations within population. Though controversy exists as to how chaos theory should be interpreted or translated into population dynamics, advancements made in the area of chaos theory are certainly allowing us a deeper insight to the world of population systems.

BIBLIOGRAPHY

"*Center for Interdisciplinary Research on Complex Systems at Northeastern University*. http://www.circs.neu.edu/

Chaos." *Exploratorium.* http://www.exploratorium.edu/complexity/lexicon/chaos.html

Costantino et al. *Nonlinear Population Dynamics.* http://vflylab.calstatela.edu/nonlin/

Costantino et al. "Population Dynamic Behavior." *Nonlinear Population Dynamics.* http://vflylab.calstatela.edu/nonlin/abstr1.htm

Elert, Glenn. *The Chaos Hypertextbook.* http://hypertextbook.com/chaos/

Frøyland, Jan. *Chaos and Coherence.* Philadelphia: Institute of Physics Publishing, 1992.

Gleick, James. *Chaos: Making a New Science.* New York: Viking, 1987.

Guastello, Stephen J. "Accidents in Health Care Settings." http://brahma.mscs.mu.edu/CTRNL/doc497b.html

Houghton, Robert. "Chapter III - System Behavior: The New Range." http://www.ceap.wcu.edu/Houghton/thesisM/Ch3.html

Ludwig et al. “Sustainability, Stability, and Resilience.”

http://www.consecol.org/Journal/vol1/iss1/art7/

Mayer-Kress, Gottfried. “What is Chaos?” http://www.santafe.edu/~gmk/MFGB/node2.html

Shibata, Tatsuo. “Logistic Map Bifurcation Diagram.”

http://chaos.c.u-tokyo.ac.jp/~shibata/Chaos/LogisticD.html

Young, T.R. *Archives: Non-linear Socio-Dynamics.*

http://www.tryoung.com/chaos/chaosindex.html